Math 347: Practice Exam 3

1. a) For a set P, state what it means for a relation R on P to be an equivalence relation.

   b) Let P be the power set of $[2n] = \{1, 2, \ldots, 2n\}$. Consider a relation R on P defined by

   $$(A, B) \in R$$

   if $A \cap [n] = B \cap [n]$, where $[n] = \{1, 2, \ldots, n\}$. Determine if R is an equivalence relation.

   c) What happens if one considers a general subset $C \subset S$ and define an equivalence relation R on S as in b).

2. If $n \in \mathbb{N}$ prove that $(n - 1)^3 + n^3 + (n + 1)^3$ is divisible by 9.

3. (a) For integers $k, n \geq 0$. Prove that

$$\sum_{i=0}^{n} \binom{i}{k} = \binom{n+1}{k+1}.$$

(b) Give a formula for the sum

$$\sum_{i=1}^{n} i^2$$

in terms of n.

4. a) For $n = 13$, what is the definition of $\mathbb{Z}/n\mathbb{Z}$? Write all of its elements.

b) For $a \in \mathbb{Z}$ let $[a] \cdot\ : \mathbb{Z}/13\mathbb{Z} \to \mathbb{Z}/13\mathbb{Z}$ be defined as

$$[a] \cdot [b] = [a \cdot b],$$

for any element $[b] \in \mathbb{Z}/13\mathbb{Z}$. Prove that this is a function.

c) How many different remainders do the numbers $\{10, 10^2, 10^3, \ldots, 10^{12}\}$ have when divided by 13?